A

RELATIONS

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Sue-Ellen wants to send a parcel to a friend. The cost of posting the parcel a fixed distance is determined by the weight of the parcel, as shown in the table.

For example, it will cost \$8.00 to post a parcel weighing at least 2 kg but less than 5 kg. It will therefore cost \$8.00 to post a parcel weighing 2 kg or 3.6 kg or 4.955 kg.

Weight w (kg)	*Cost* \$$C$
$1 \leqslant w < 2$	\$5.00
$2 \leqslant w < 5$	\$8.00
$5 \leqslant w < 10$	\$12.00
$10 \leqslant w < 15$	\$16.00
$15 \leqslant w < 20$	\$20.00

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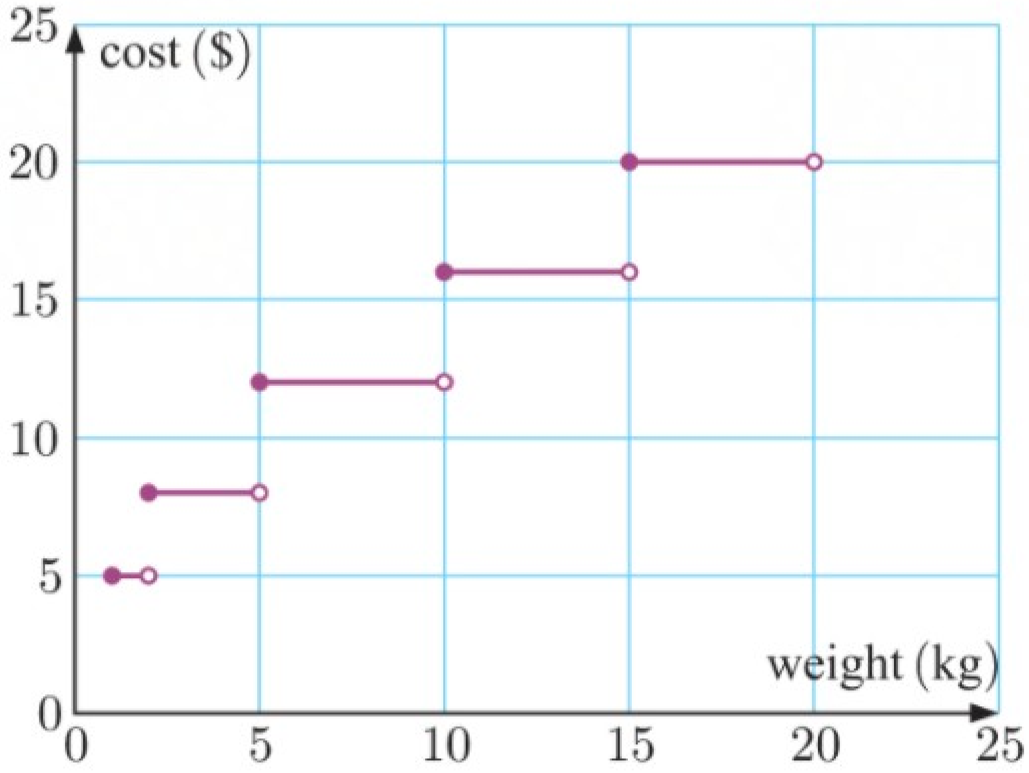
We can illustrate the postal charges on a graph.

An end point that is included has a filled in circle.

An end point that is not included has an open circle.

There is a *relationship* between the variables *weight* and *cost*, so the table of costs is an example of a **relation**.



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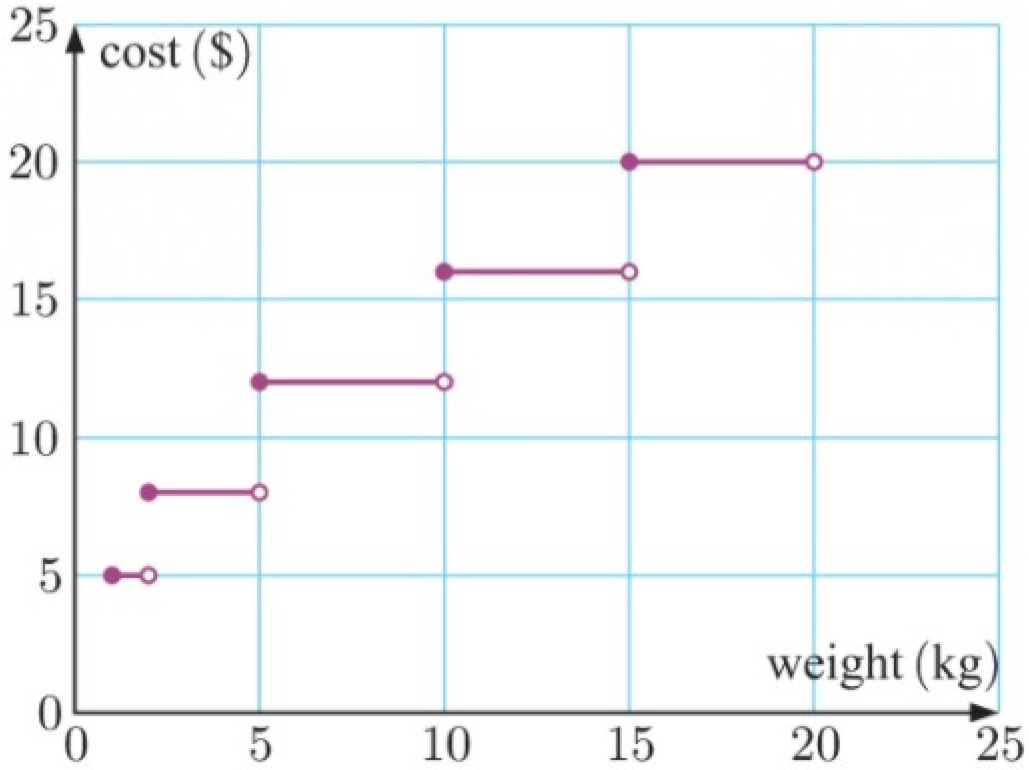
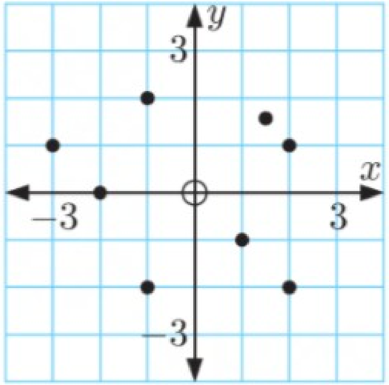


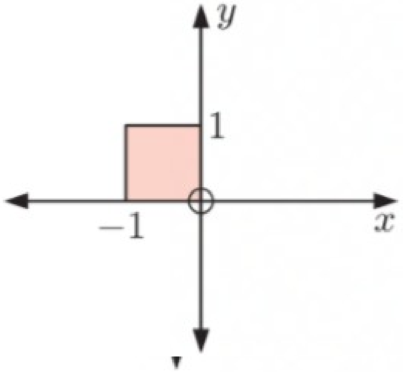
A relation may be a finite number of ordered pairs, for example $\{(2, 8), (3, 8), (4, 8), (5, 12)\}$, or an infinite number of ordered pairs, such as the relation between the variables *weight* and *cost* in the postal charges above.

A **relation** is any set of points which connects two variables.

The following are examples of relations:

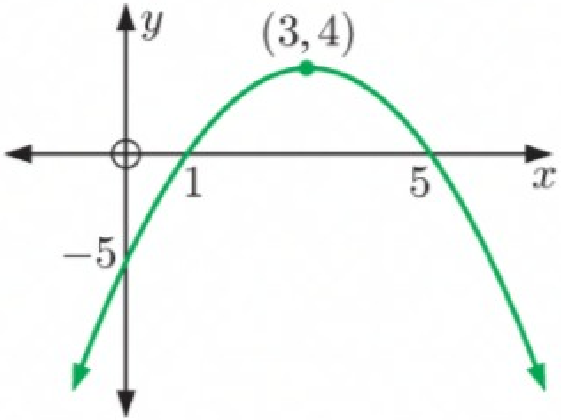
- The set of 8 points represented by the dots is a relation. There is no equation connecting the variables x and y in this case.



- 

  The set of all points on and within the illustrated square is a relation. It is the set of all points (x, y) such that $-1 \leqslant x \leqslant 0$ and $0 \leqslant y \leqslant 1$.

- The set of all points on this parabola is a relation. It is the set of all points (x, y) lying on the curve $y = -x^2 + 6x - 5$.

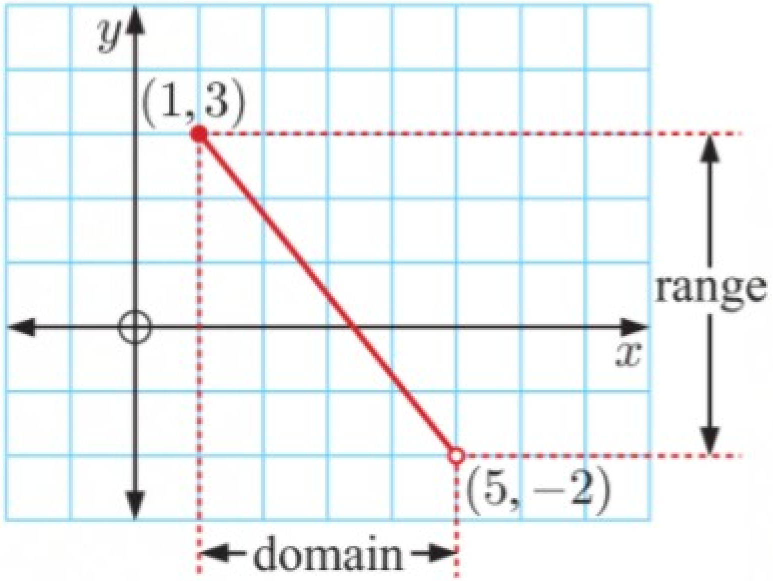


DOMAIN AND RANGE

The **domain** of a relation is the set of possible values that x may have.

The **range** of a relation is the set of possible values that y may have.

The domain and range of a relation are often described using **interval notation**.

- 

The domain consists of all real x such that $1 \leqslant x < 5$. We write this as:

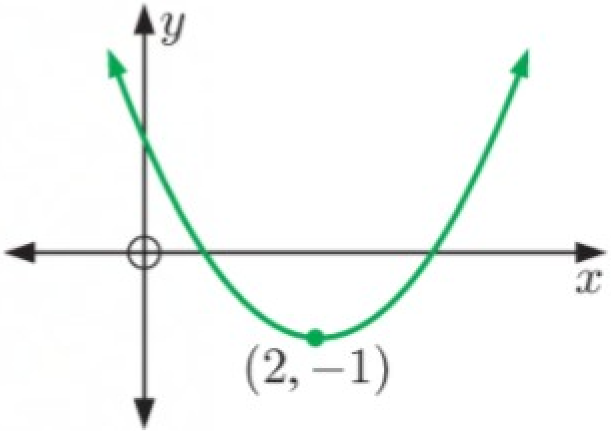
$$\underset{\substack{\uparrow\\\text{the set of all}}}{\{}x \underset{\substack{\nwarrow\\\text{such that}}}{|} 1 \leqslant x < 5\}.$$

The range is $\{y \mid -2 < y \leqslant 3\}$.

● indicates the point is included.

○ indicates the point is excluded.

$\mathbb{R}$ represents the set of all real numbers, or all numbers on the number line.

- 

The domain is $\{x \mid x \in \mathbb{R}\}$.
The range is $\{y \mid y \geqslant -1\}$.

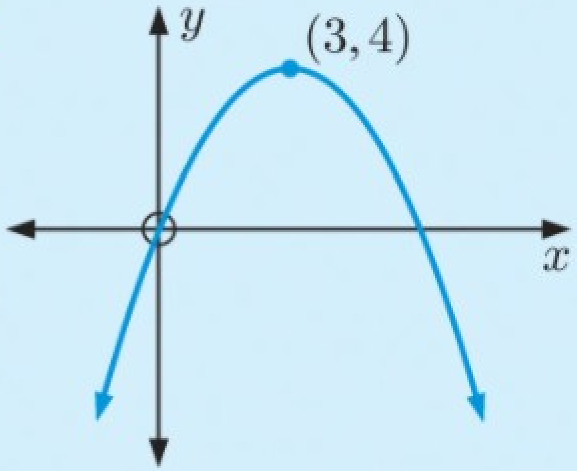
● indicates the point is included.
○ indicates the point is excluded.

$\mathbb{R}$ represents the set of all real numbers, or all numbers on the number line.

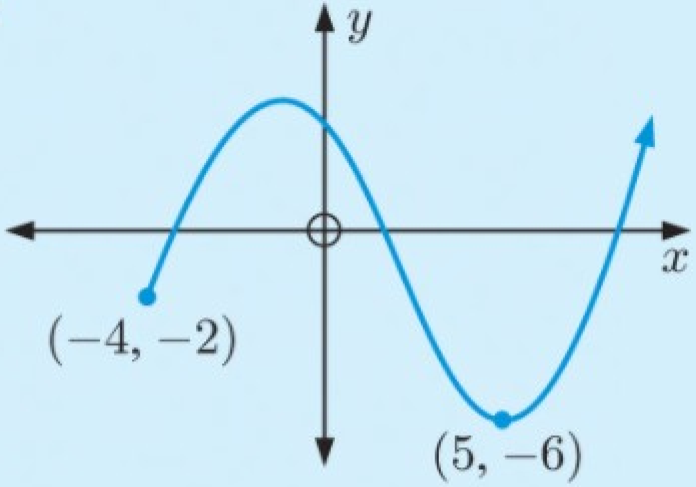
Example 1 **Self Tutor**

For each of the following graphs, state the domain and range:

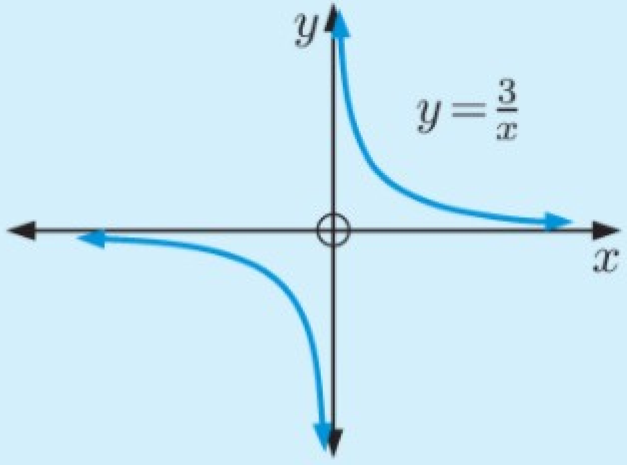
a



b



c



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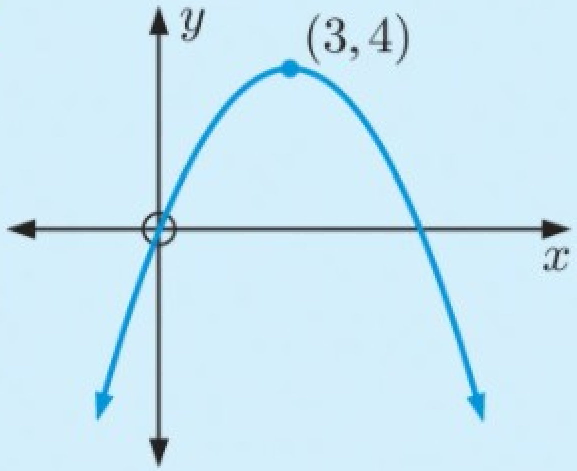
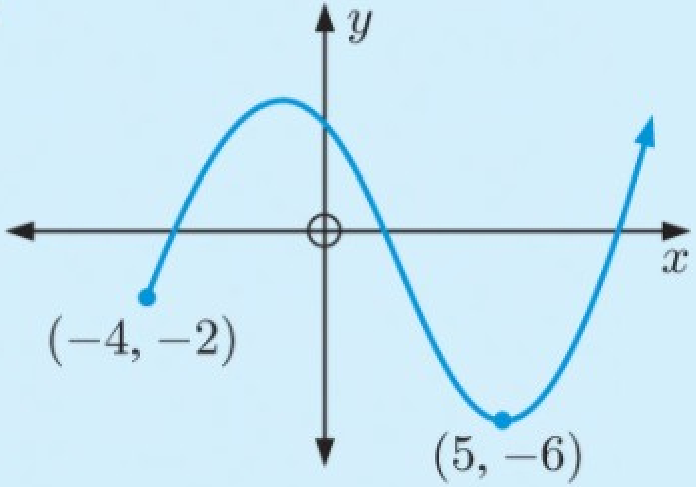
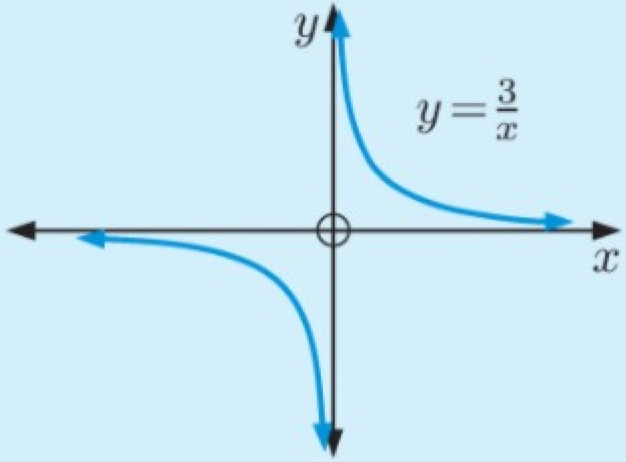


b



c



a Domain is $\{x \mid x \in \mathbb{R}\}$.
Range is $\{y \mid y \leqslant 4\}$.

b Domain is $\{x \mid x \geqslant -4\}$.
Range is $\{y \mid y \geqslant -6\}$.

c Domain is $\{x \mid x \neq 0\}$.
Range is $\{y \mid y \neq 0\}$.

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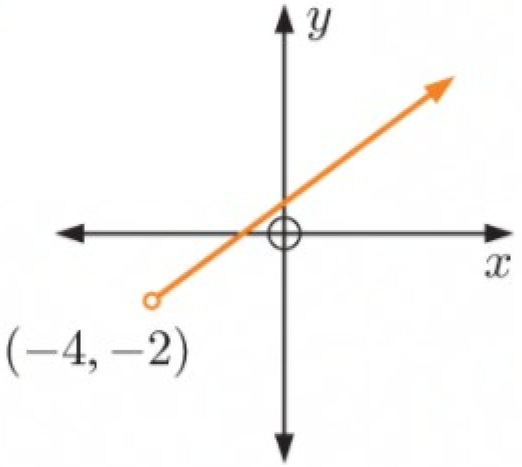
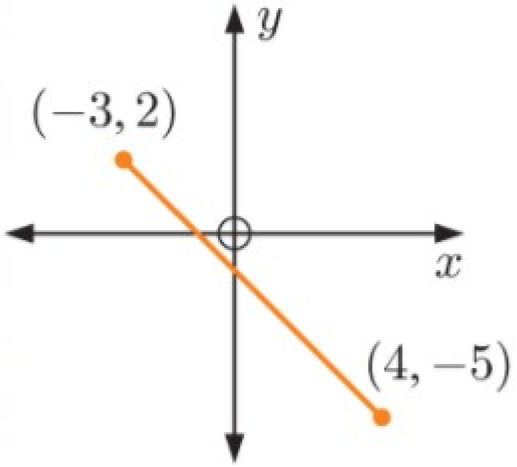
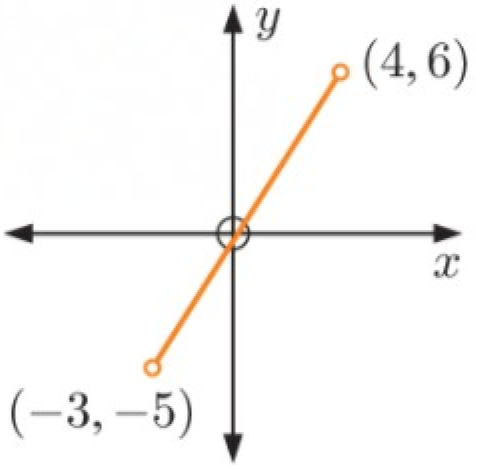


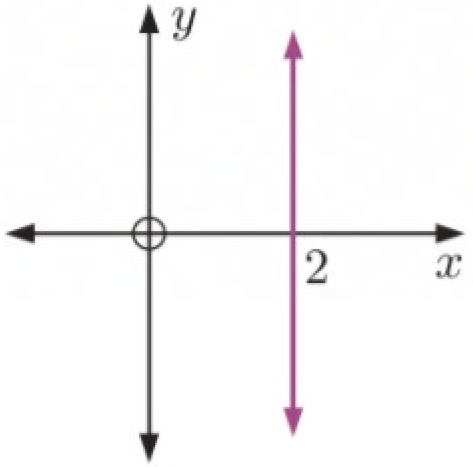
b



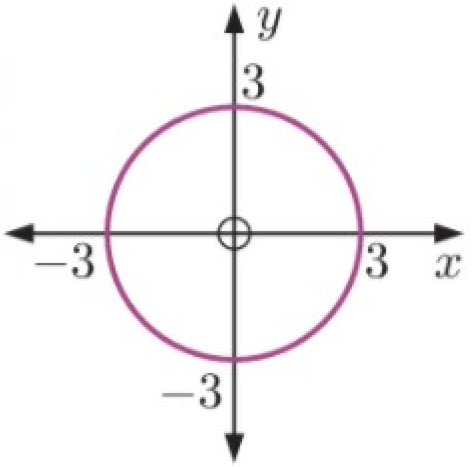
c



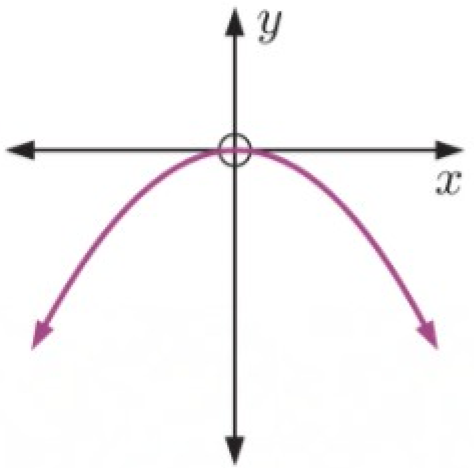
d



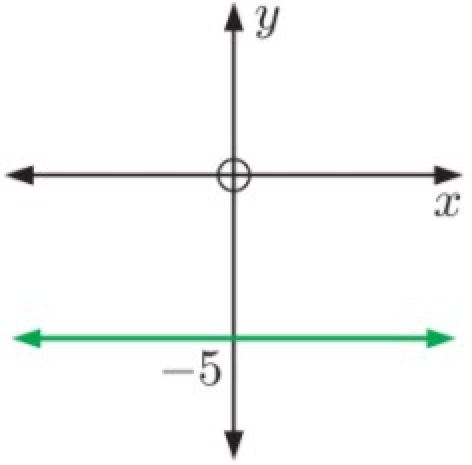
e



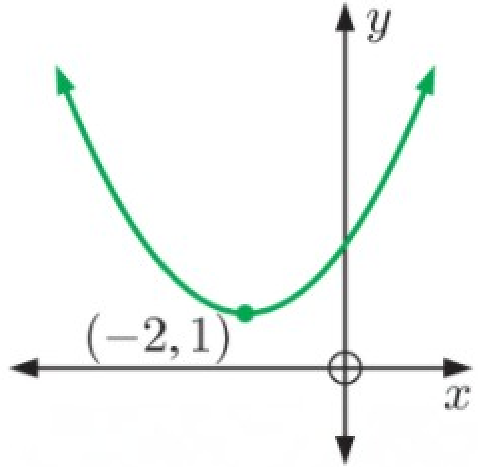
f



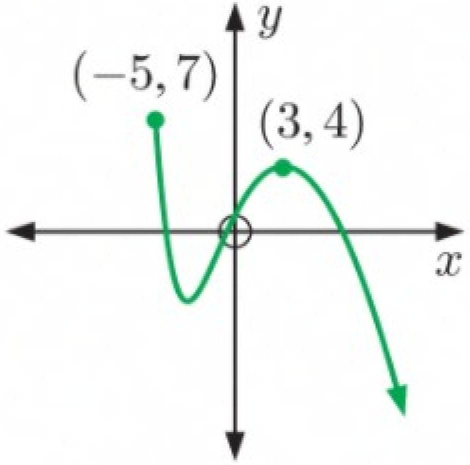
g



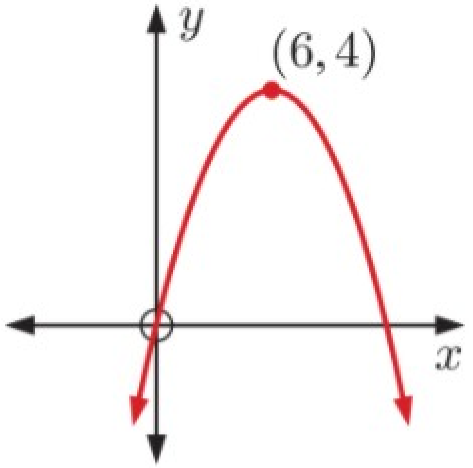
h



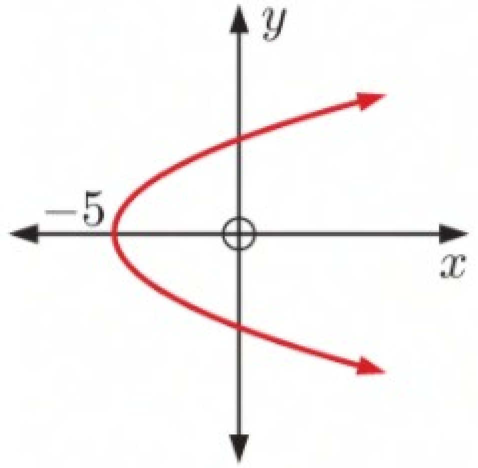
i



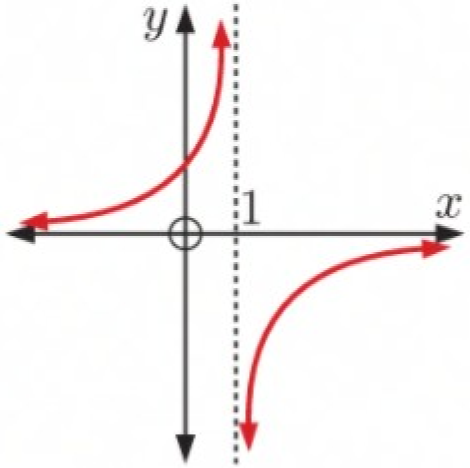
j



k

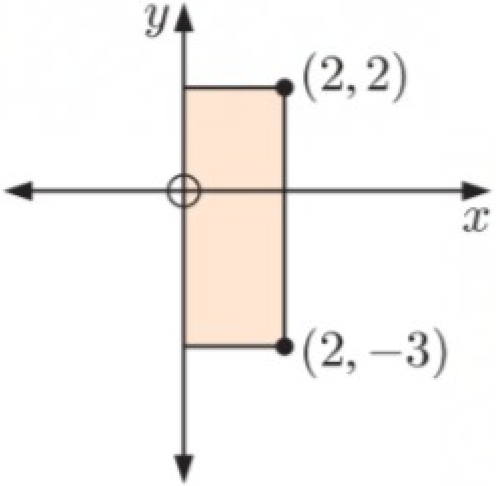


l

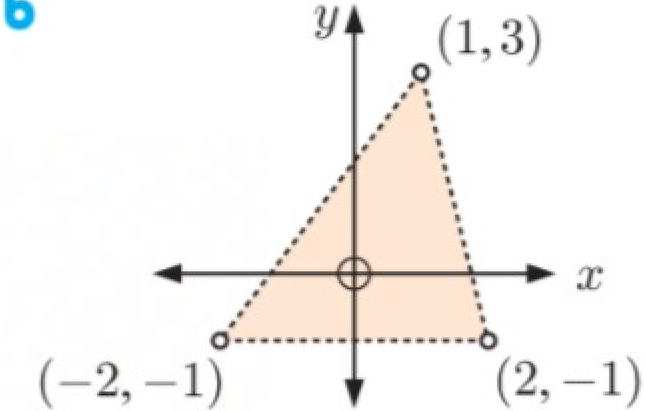


2 State the domain and range of:

a

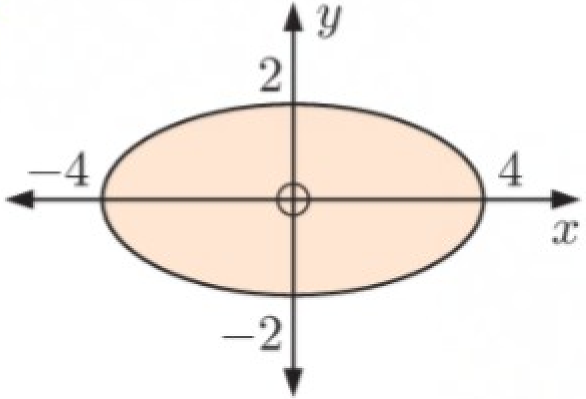


b



(boundaries not included)

c



EXERCISE 15A

1 **a** Domain is $\{x \mid x > -4\}$. Range is $\{y \mid y > -2\}$.

b Domain is $\{x \mid -3 \leqslant x \leqslant 4\}$. Range is $\{y \mid -5 \leqslant y \leqslant 2\}$.

c Domain is $\{x \mid -3 < x < 4\}$. Range is $\{y \mid -5 < y < 6\}$.

d Domain is $\{x \mid x = 2\}$. Range is $\{y \mid y \in \mathbb{R}\}$.

e Domain is $\{x \mid -3 \leqslant x \leqslant 3\}$. Range is $\{y \mid -3 \leqslant y \leqslant 3\}$.

f Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \leqslant 0\}$.

g Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y = -5\}$.

h Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \geqslant 1\}$.

i Domain is $\{x \mid x \geqslant -5\}$. Range is $\{y \mid y \leqslant 7\}$.

j Domain is $\{x \mid x \in \mathbb{R}\}$. Range is $\{y \mid y \leqslant 4\}$.

k Domain is $\{x \mid x \geqslant -5\}$. Range is $\{y \mid y \in \mathbb{R}\}$.

l Domain is $\{x \mid x \in \mathbb{R},\ x \neq 1\}$.
Range is $\{y \mid y \in \mathbb{R},\ y \neq 0\}$.

2 **a** Domain is $\{x \mid 0 \leqslant x \leqslant 2\}$. Range is $\{y \mid -3 \leqslant y \leqslant 2\}$.

b Domain is $\{x \mid -2 < x < 2\}$. Range is $\{y \mid -1 < y < 3\}$.

c Domain is $\{x \mid -4 \leqslant x \leqslant 4\}$. Range is $\{y \mid -2 \leqslant y \leqslant 2\}$.

B FUNCTIONS

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A **function** is a relation in which no two different ordered pairs have the same first member.

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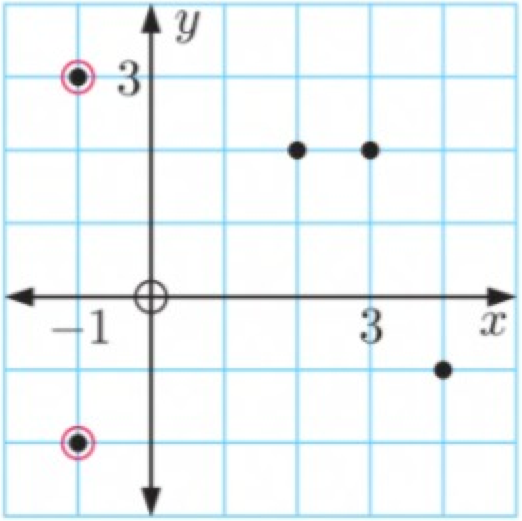
The grid alongside shows the set of points:

$$\{(-1, 3), (2, 2), (-1, -2), (3, 2), (4, -1)\}.$$

The two circled points $(-1, -2)$ and $(-1, 3)$ have the same first member, so the set of points is a relation but not a function.



GEOMETRIC TEST FOR FUNCTIONS: "VERTICAL LINE TEST"

Suppose we draw all possible vertical lines on the graph of a relation.

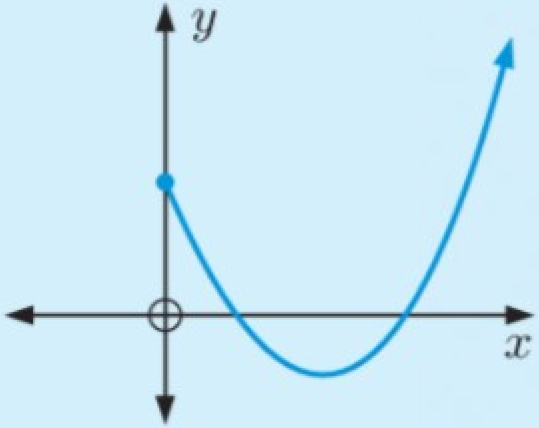
- If each line cuts the graph at most once, then the relation is a function.
- If *any* line cuts the graph more than once, then the relation is *not* a function.

Example 2

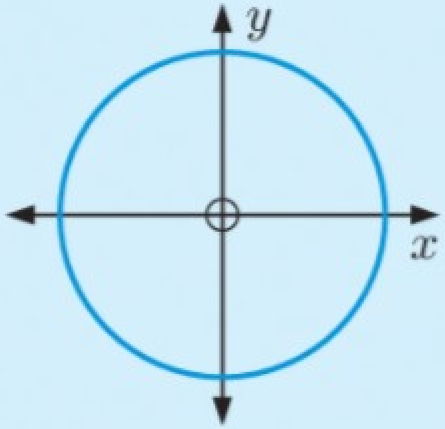
Self Tutor

Which of these relations are functions?

a



b



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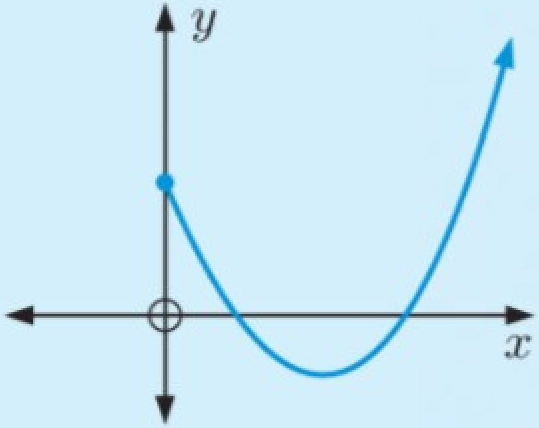
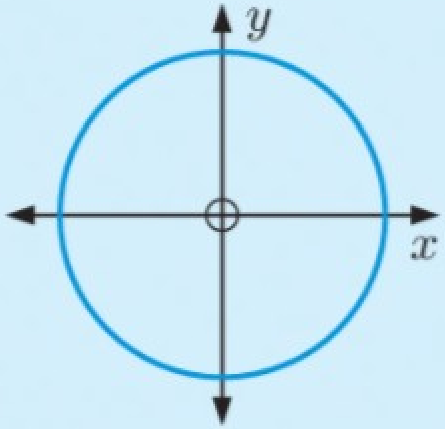
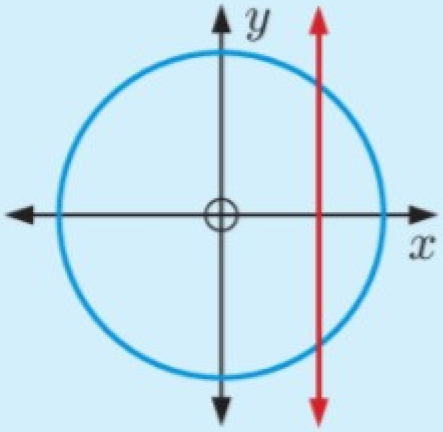


b



a Every vertical line we could draw cuts the graph only once.
$\therefore$ the relation is a function.

b This vertical line cuts the graph twice.
$\therefore$ the relation is not a function.



EXERCISE 15B

1 Which of the following sets of ordered pairs are functions? Give reasons for your answers.

a $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

b $\{(-1, 2), (-3, 2), (3, 2), (1, 2)\}$

c $\{(2, 5), (-1, 4), (-3, 7), (2, -3)\}$

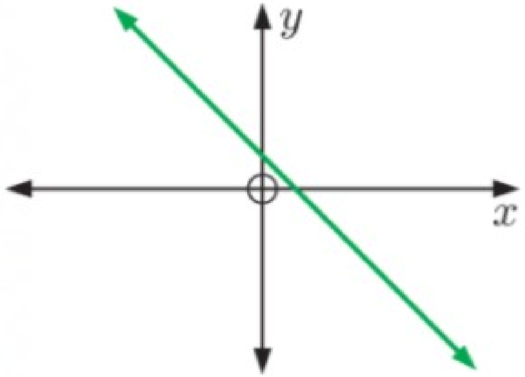
d $\{(3, -2), (3, 0), (3, 2), (3, 4)\}$

e $\{(-7, 0), (-5, 0), (-3, 0), (-1, 0)\}$

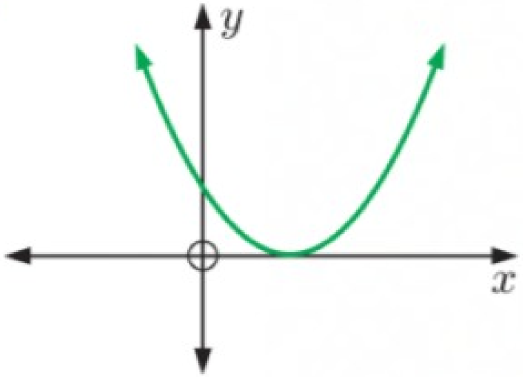
f $\{(0, 5), (0, 1), (2, 1), (2, -5)\}$

2 Use the vertical line test to determine which of the following relations are functions:

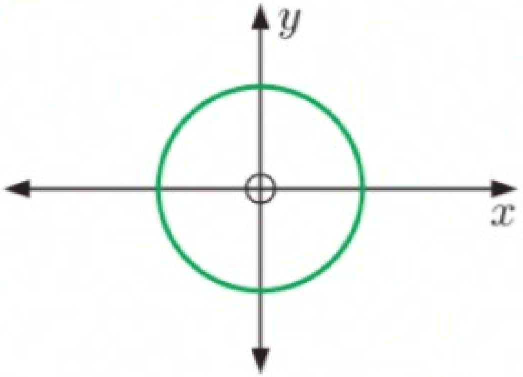
a



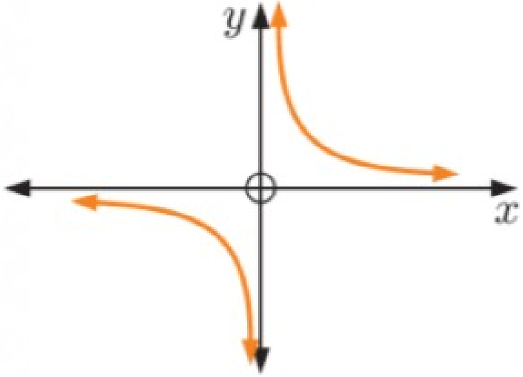
b



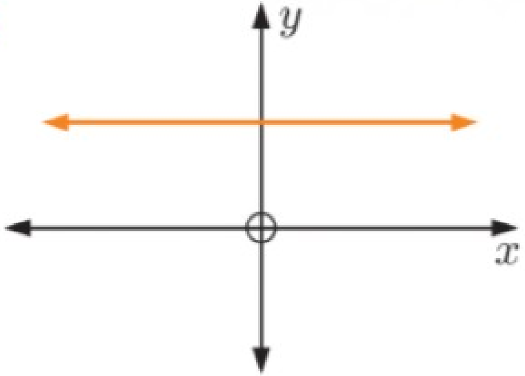
c



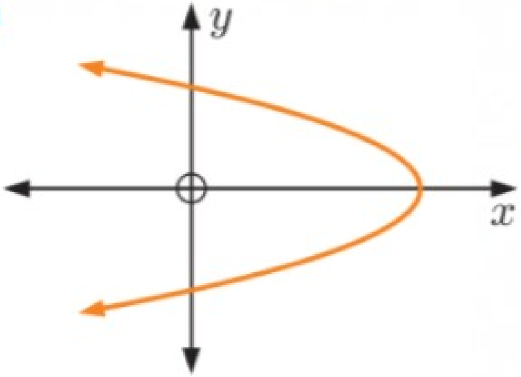
d



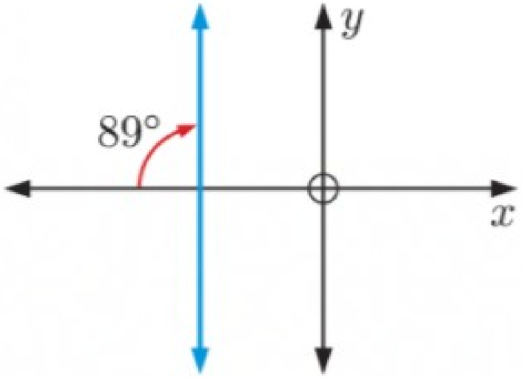
e



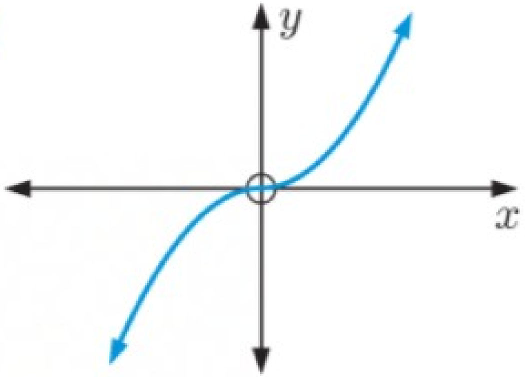
f



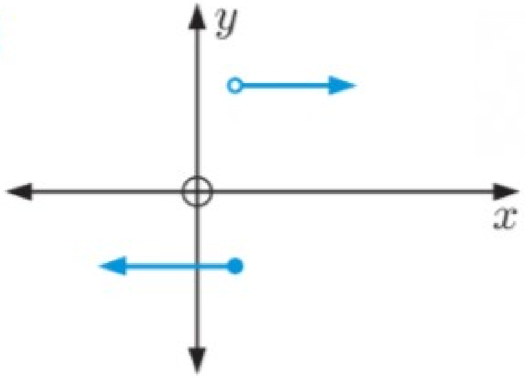
g



h



i



3 Will the graph of a straight line always be a function? Explain your answer.

EXERCISE 15B

1 **a**, **b**, and **e** are functions as no two ordered pairs have the same x-coordinate.

2 **a**, **b**, **d**, **e**, **g**, **h**, and **i** are functions.

3 No, a vertical line is not a function as it does not satisfy the vertical line test.